

Minimal SUSY SO(10), b - τ Unification and Large Neutrino Mixings

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Abstract

We show that the assumption of type II seesaw mechanism for small neutrino masses coupled with $b - \tau$ mass unification in a minimal SUSY SO(10) model leads not only to a natural understanding of large atmospheric mixing angle (θ_{23}) among neutrinos, as recently pointed out, but also to large solar angle (θ_{12}) and a small $\theta_{13} \equiv U_{e3}$ as required to fit observations. This is therefore a minimal, completely realistic grand unified model for all low energy observations that naturally explains the diverse mixing patterns between the quark and leptons without any additional inputs such as extra global symmetries. The proposed long baseline neutrino experiments will provide a crucial test of this model since it predicts $U_{e3} \simeq 0.16$ for the allowed range of parameters.

I. INTRODUCTION

The various neutrino oscillation experiments such as those involving solar and atmospheric neutrinos as well as the KEK and KAMLAND experiments that involve laboratory produced neutrinos have now produced quite convincing evidence that neutrinos have mass and they mix among themselves. Although the neutrinos have a great deal of similarity with quarks as far as the weak interactions go, the oscillation results have revealed a profound difference i.e. two of the three neutrino mixings are very large whereas all quark mixing angles are very small. Understanding this difference is a major challenge for theoretical particle physics today. The problem becomes particularly acute in models that unify quarks and leptons such as the SO(10) grand unified models [1], which are considered as prime candidates for describing neutrino masses.

Some of the reasons that make SO(10) models so attractive as grand unification theories of nature are the following: (i) in SO(10) model, all fermions can be part of a single spinor representation; (ii) it contains the left-right symmetric unification group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ [2] which provides a more satisfactory way to understand the origin of parity violation in Nature and (iii) finally, perhaps the single most important reason is the natural understanding of small neutrino masses via the seesaw mechanism [3] since the single spinor representation discussed above that contains all the standard model fermions also contains the right handed neutrino, needed in implementing the seesaw mechanism.

A closer look at the details of neutrino oscillation data in fact provides one more compelling reason for the SO(10) model: in order to understand the atmospheric neutrino data, we need the heaviest neutrino mass to be larger than 0.05 eV. The seesaw formula i.e. $m_\nu \sim \frac{m_D^2}{M_R}$ (where m_D is the neutrino Dirac mass and M_R is the mass of the right handed neutrino contributing to the neutrino mass needed to understand the atmospheric data) then tells us that there must be one right handed $M_R \leq 10^{15}$ GeV. This value is considerably smaller than the Planck mass and therefore one is faced with a new hierarchy problem similar to the corresponding problem of the standard model. However, it was pointed out long ago [4] that the Majorana mass of the RH neutrino owes its origin to the breaking of local B-L symmetry which implies that $M_R \simeq M_{B-L}$. Local B-L symmetry therefore provides a natural way to understand the smallness of the RH neutrino mass compared to M_{Pl} . What is very interesting is that SO(10) group also contains the local B-L as a subgroup.

SO(10) model, despite its attractiveness for understanding the overall scale of neutrino masses, runs into a potential trouble in providing an understanding of the observed mixings. The problem arises from the fact that SO(10) also contains the quark-lepton unification $SU(4)_c$ group of Pati and Salam, which in the simplest approximation leads to equal quark and lepton mixing angles and one needs to make further assumptions to get a handle on the mixings [5]. An obvious conceptual problem is that if one of these models is ruled out by data, one would not be able to tell whether it is the SO(10) unification which is “at fault” or it is one of the assumptions used to derive neutrino mixings.

A different approach to this issue was taken in ref. [6]. The idea was to avoid the use of any symmetries beyond the gauge symmetry, in this case SO(10) and use the minimal set of Higgs fields that can break the group down to the standard model and give mass to the fermions. It turns out that if we choose Higgs fields in **10** and **$\overline{126}$** , the Yukawa superpotential contains enough parameters to fit the observed fermion masses and mixings

of the standard model. It was observed in ref. [6], that in this model the neutrino masses and mixings are completely predicted upto an overall scale, when one uses the seesaw mechanism which is part of the SO(10) model¹. To break the gauge symmetry fully, two additional Higgs multiplets belonging to **45**+**54** are included. They do not contribute to the fermion masses, leaving the conclusions on neutrino masses unchanged.

An additional appeal of breaking B-L symmetry of SUSY SO(10) by an **126**, as opposed to by **16** Higgs, is that it automatically leaves R-parity as an exact symmetry and thereby explains, why neutralino is a stable dark matter [7,8]. This is because the submultiplet of **126** that breaks B-L carries B-L =2. Therefore R-parity (defined by $Rp = (-1)^{3(B-L)+2S}$) quantum number of this field is even and therefore, its vev leaves R-parity unbroken. In contrast in models where B-L is broken by a **16**-plet of Higgs, the B-L symmetry is broken by one unit and without any additional symmetries (e.g. matter parity), neutralino is unstable and cannot therefore serve as a dark matter. Of course, if a fundamental theory e.g. a superstring theory that led to an SO(10) model with appropriate additional symmetries that guarantee the stability of neutralino was known, then the above objection to an **16** Higgs would not apply.

The initial analyses of neutrino mixings within the framework of Ref. [6] used the simple seesaw formula (type I seesaw) and are now in disagreement with data. In subsequent papers [9–13], this idea has been analysed (in some cases by including more than one **10** Higgses) to see how close one can come close to the observed neutrino parameters. The conclusion now appears to be that one needs CP violating phases to achieve this goal, as noted in [13].

A way out of this problem is to use the type II seesaw mechanism, as was initially done in [11], where an induced triplet vev is added to the usual type I seesaw formula arising from the RH neutrino intermediate state. In models which have asymptotic parity symmetry such as left-right or SO(10) models, type II seesaw arises if both parity and B-L symmetry are broken at the same scale.

A very interesting point about this approach has been noted in a recent paper [14], where it has been shown that if we restrict ourselves to the 2-3 sector of the model and use the type II seesaw mechanism, then the $b - \tau$ unification of supersymmetric grand unified theories leads to a neutrino Majorana mass matrix which explains the large $\nu_\mu - \nu_\tau$ mixing angle needed to understand atmospheric neutrino data. The important point is that no symmetries are needed to get this result.

To see how this result is very generic to SO(10) models with **126** vev, note that: (i) in SO(10) model, the neutrino mass matrix is given by the type II seesaw formula, with two contributions: one coming from the righthanded neutrino intermediate state and another coming from an induced triplet vev generic to these models, as already mentioned; and (ii) that for certain range of parameters, the induced triplet vev term can dominate the neutrino mass matrix. As was shown in [11], under these circumstances, one gets a sumrule

$$\mathcal{M}_\nu = a(\mathcal{M}_\ell - \mathcal{M}_d) \quad (1)$$

¹This is to be contrasted with the SU(5) case where the minimal Higgs set needed to break the gauge symmetry i.e. **5**+**24** Higgses lead to the mass relation $m_e/m_\mu = m_d/m_s$ that is in contradiction with observations.

In [14], it was observed that since this relation is valid at the seesaw scale, one must use the extrapolated quark and lepton masses in the formula. The fact that at or near the GUT scale $m_b/m_\tau \simeq 1 - 1.2$ depending on the value of $\tan\beta$, implies that the 3-3 element of the \mathcal{M}_ν which is proportional to $m_b - m_\tau$ is of the same order as the off diagonal elements of the \mathcal{M}_{nu} in the 2-3 subsector leading to the largeness of the atmospheric mixing angle without any further assumptions.

It is however essential to do a complete three generation analysis of this model if this important observation is to lead to a realistic SO(10) model for understanding all neutrino mixings. In fact, since the model has no free parameters, it is a priori not obvious that within this framework one would simultaneously get a large solar mixing angle and a small $\theta_{13} \equiv U_{e3}$ as well as the correct value for the ratio $\Delta m_\odot^2/\Delta m_A^2$. It is the goal of this paper to analyze this question.

We find that for a narrow range of the quark masses, the model does indeed lead to the correct mass difference-squares as well as large mixing angles θ_{12} and θ_{23} needed to understand the solar and atmospheric neutrino oscillations while at the same time keeping $U_{e3} \leq 0.16$ as required by the reactor data. We find that for this entire range, the reactor angle $U_{e3} \simeq 0.16$ providing a clear way to test the model in the proposed long base line neutrino experiments.

This paper is organized as follows: in sec. II, we review the basic equations of the model for the supersymmetric case; in sec. III, we explain the conditions under which the induced triplet vev dominates neutrino masses; in sec. IV, we describe our method for solving the equations to predict the neutrino parameters. In sec. V, we give our results for the neutrino masses and the mixing angles. In sec. VI, we give our conclusions. In this paper, we ignore the CP violating phases.

II. THE MASS SUMRULES FOR MINIMAL SO(10)

We consider supersymmetric SO(10) group with the Higgs fields belonging to the representations **45**+**54** for breaking SO(10) group down to the left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ and the minimal Higgs set **10**+**126**+**126** that couple to matter and also break the $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ group down to $SU(3)_c \times U(1)_{em}$. It is the latter set i.e. **10** \oplus **126** which is crucial to our discussion of fermion masses. The first stage of the symmetry breaking therefore could have been accomplished by alternative Higgs multiplets e.g. by **210** of Higgs without effecting our results. As has been noted earlier [6,10], the set **10**+**126** which couple to matter contain two pairs of MSSM Higgs doublets belonging to (2,2,1) and (2,2,15) submultiplets (under $SU(2)_L \times SU(2)_R \times SU(4)_c$ subgroup of SO(10)). We denote the two pairs by $\phi_{u,d}$ and $\Delta_{u,d}$. At the GUT scale, by some doublet-triplet splitting mechanism these two pairs reduce to the MSSM Higgs pair (H_u, H_d) , which can be expressed in terms of the ϕ and Δ as follows:

$$\begin{aligned} H_u &= \cos\alpha_u \phi_u + \sin\alpha_u \Delta_u \\ H_d &= \cos\alpha_d \phi_d + \sin\alpha_d \Delta_d \end{aligned} \tag{2}$$

The details of the doublet-triplet splitting mechanism that leads to the above equation are not relevant for what follows and we do not discuss it here. As in the case of MSSM, we will assume that the Higgs doublets $H_{u,d}$ have the vevs $\langle H_u^0 \rangle = v \sin\beta$ and $\langle H_d^0 \rangle = v \cos\beta$.

In orders to discuss fermion masses in this model, we start with the SO(10) invariant superpotential giving the Yukawa couplings of the **16** dimensional matter spinor ψ_i (where i, j denote generations) with the Higgs fields $H_{10} \equiv \mathbf{10}$ and $\Delta \equiv \overline{\mathbf{126}}$.

$$W_Y = h_{ij}\psi_i\psi_j H_{10} + f_{ij}\psi_i\psi_j \Delta \quad (3)$$

SO(10) invariance implies that h and f are symmetric matrices. We ignore the small effects coming from the higher dimensional operators. Below the B-L breaking (seesaw) scale, we can write the superpotential terms for the charged fermion Yukawa couplings as:

$$W_0 = h_u Q H_u u^c + h_d Q H_d d^c + h_e L H_d e^c + \mu H_u H_d \quad (4)$$

where

$$\begin{aligned} h_u &= h \cos \alpha_u + f \sin \alpha_u \\ h_d &= h \cos \alpha_d + f \sin \alpha_d \\ h_e &= h \cos \alpha_d - 3f \sin \alpha_d \end{aligned} \quad (5)$$

In general $\alpha_u \neq \alpha_d$ and this difference is responsible for nonzero CKM mixing angles. In terms of the GUT scale Yukawa couplings, one can write the fermion mass matrices at the seesaw scale as:

$$\begin{aligned} M_u &= (\bar{h} + \bar{f}) \\ M_d &= (\bar{h} r_1 + \bar{f} r_2) \\ M_e &= (\bar{h} r_1 - 3\bar{f} r_2) \\ M_{\nu D} &= (\bar{h} - 3\bar{f}) \end{aligned} \quad (6)$$

where

$$\begin{aligned} \bar{h} &= h \cos \alpha_u \sin \beta \\ \bar{f} &= f \sin \alpha_u \sin \beta \\ r_1 &= \frac{\cos \alpha_d}{\cos \alpha_u} \cot \beta \\ r_2 &= \frac{\sin \alpha_d}{\sin \alpha_u} \cot \beta \end{aligned} \quad (7)$$

To count the number of parameters describing the fermion sector, we first ignore CP phases and choose a basis where \bar{h} is diagonal. Since \bar{f} is symmetric, we have a total of nine parameters from the couplings and including $\alpha_{u,d}$ and β gives us a total of twelve parameters. All these parameters can be determined by fitting the the six quark masses, three lepton masses and three CKM angles. This enables a complete determination of the neutrino masses upto an overall scale related to the B-L symmetry breaking and the three mixing angles. The model is therefore completely predictive in the neutrino sector.

In order to determine the neutrino masses and mixings, one uses the seesaw mechanism. As noted in the introduction, most previous works on this model except the works in Ref. [11,14] used the type one seesaw mechanism where the neutrino mass matrix is given by the formula:

$$\mathcal{M}_\nu = -M_{\nu D} M_{N_R}^{-1} M_{\nu D}^T \quad (8)$$

where $M_{N_R} = f v_{B-L}$. On the other hand it is well known that in asymptotically parity conserving theories including SO(10), the true seesaw formula [15] (called type II here and in literature) has a second term which arises from an induced $SU(2)_L$ triplet vev or from higher dimensional terms involving left doublets:

$$\mathcal{M}_\nu = \bar{f} \sigma_L - M_{\nu D} M_{N_R}^{-1} M_{\nu D}^T \quad (9)$$

where $\sigma_L = \lambda \frac{v^2}{v_{B-L}}$, where v is the $SU(2)_L$ breaking scale and λ is a combination of parameters in the Higgs potential. The type II seesaw formula has two new parameters σ_L and v_{B-L} instead of one in the type I case; but for different ranges of initial parameters in Higgs potential, either the first or the second term can be made to dominate [16]. We will work within the assumption that it is the first term that dominates the seesaw formula [14]. This situation can arise when $\sigma_L \gg v^2/v_{B-L}$ (or λ which is a ratio of scalar coupling parameters is much larger than one). We elaborate the circumstances when this happens in sec. III.

It was noted in several papers [6,10–12] that if one uses type I seesaw, this model cannot produce two large neutrino mixing angles if CP phases are ignored. It has been shown recently [13] that once the CP phases are included, one can get bi-large neutrino mixing pattern. But here we search for two large mixing angle solutions without invoking the CP phase. As noted in [14], if one uses type II seesaw and assumes further that the first term dominates, large atmospheric mixing angle follows naturally as a consequence of b- τ unification. Whether this also simultaneously yields a large solar mixing angle and a small U_{e3} remained an open question. In this present paper we present a detailed analysis of this idea in a full three generation model and show that for a very narrow range of quark masses, this model also predicts large solar mixing angle as well as the correct solar mass splitting together with a small U_{e3} . Since the model has hierarchical mass pattern, the atmospheric mass difference-squared is related to the overall scale of the triplet vev in the type II seesaw formula and cannot be predicted.

III. DOMINANCE OF INDUCED TRIPLET VEV

In this section, we would like to discuss the parameter range where the induced triplet vev term dominates the neutrino mass matrix. To elucidate this let us discuss the origin of the triplet vev in our minimal SO(10) model. First we note the decomposition of the **126** under the group $SU(2)_L \times SU(2)_R \times SU(4)_c$:

$$\overline{\mathbf{126}} = (1, 1, 6) \oplus (2, 2, 15) \oplus (3, 1, \bar{10}) \oplus (1, 3, 10) \quad (10)$$

The $SU(2)_L$ triplet that contributes to the type II seesaw formula is contained in the multiplet $\Delta_L \equiv (3, 1, \bar{10})$ and it couples to the left handed multiplet $\psi \equiv (2, 1, 4)$ of the **16** dimensional SO(10) spinor that contains the matter fermions i.e. $\psi_L \psi_L \Delta_L$. On the other hand the mass of the RH neutrinos comes from the coupling of $\Delta_R \equiv (1, 3, 10)$ submultiplet of **126** to the right handed fermion multiplet $\psi_L^c \equiv (1, 2, \bar{4})$ i.e. $\psi_L^c \psi_L^c \Delta_R$.

The vev of the neutral member of Δ_R breaks the B-L symmetry and gives mass to the RH neutrinos and generates the second term in the type II seesaw formula. To see how the Δ_L^0 vev arises, note that the general superpotential of the model contains terms of type $\lambda_1 \mathbf{126}^2 \cdot \mathbf{54}$ and $\lambda_2 \mathbf{10} \cdot \mathbf{10} \cdot \mathbf{54}$. In the Higgs potential, this generates a term (from $|F_{\mathbf{54}}|^2$) of the form $\mathbf{10} \cdot \mathbf{10} \cdot \overline{\mathbf{126}} \cdot \overline{\mathbf{126}}$. In this term, there is a term of the form $\phi(2, 2, 1)^2 \Delta_L(3, 1, 10) \Delta_R(1, 3, \bar{10})$ with a coefficient $\lambda_1 \lambda_2$. Furthermore, in the Higgs potential, there is a mass term for $\Delta_L(3, 1, 10)$ of the form $\mu_\Delta^2 + \lambda_3 v_U^2$, where v_U is the GUT scale. On minimizing the potential, these two terms lead to a vev for the $SU(2)_L$ triplet $\sigma_L \equiv \langle \Delta_L^0 \rangle \simeq \frac{\lambda_1 \lambda_2 v_{wk}^2 v_{B-L}}{\mu_\Delta^2 + \lambda_3 v_U^2}$. It is now clear that if we choose λ_3 such that $\mu_\Delta^2 + \lambda_3 v_U^2 \ll v_{B-L}^2$, then the entries in the second matrix in the type II seesaw formula can much smaller than σ_L and Eq.1 holds. We will work in the domain of the parameter space where this happens.

If the triplet vev contribution to the neutrino mass matrix dominates in the type II seesaw formula, then Equation (6) can be used to derive the sumrule

$$\mathcal{M}_\nu = a(M_\ell - M_d) \quad (11)$$

Using this equation in second and third generation sector, one can understand the results of [14] in a heuristic manner as follows. The known hierarchical structure of quark and lepton masses as well as the known small mixings for quarks suggest that $M_{\ell,d}$ have the following pattern

$$M_\ell \simeq m_\tau \begin{pmatrix} \epsilon_{\ell,1} & \epsilon_{\ell,2} \\ \epsilon_{\ell,2} & 1 \end{pmatrix} \quad (12)$$

and

$$M_d \simeq m_b \begin{pmatrix} \epsilon_{d,1} & \epsilon_{d,2} \\ \epsilon_{d,2} & 1 \end{pmatrix} \quad (13)$$

where $\epsilon_{\ell,d,i} \ll 1$. It is then clear that if there is approximate $b - \tau$ unification as it appears to be the case if the theory below the B-L breaking scale is MSSM, then in $M_\ell - M_d$ matrix, there is a high degree of cancellation in the 33 entry making this entry comparable to all the other entries in this matrix. The atmospheric mixing angle which is given by $\tan 2\theta_A \simeq (m_\tau \epsilon_{\ell,2} - m_b \epsilon_{d,2}) / (m_b - m_\tau + m_b \epsilon_{d,1} - m_\tau \epsilon_{\ell,1})$ becomes very large at the B-L breaking scale. Since renormalization played an important role in obtaining this result, one must ask what happens to the neutrino mixings once they are extrapolated to the weak scale [17]. It is well known [17] that for the case of normal hierarchy for neutrino masses as is the case here, the MSSM RGE's do not change the mixing angles very much and the seesaw scale result persists at the weak scale with only minor changes.

IV. DETAILS OF CALCULATION

In this section, we outline our method for determining the neutrino mixing parameters. For this purpose, we first note that the matrices \bar{h} and \bar{f} in Eq. (6) can be eliminated in terms the mass matrices $M_{u,d}$ so that we have a sumrule involving the three mass matrices $M_{u,d,\ell}$. Before giving the sum rule, we note that we will work in a basis where M_d is diagonal and is given by $M_u = V^T \cdot M_u^D \cdot V$ (where M_u^D is the diagonal mass matrix of up type quark

and V is the CKM mixing matrix). This can be done without any loss of generality. We also introduce a new set of matrices $\tilde{M}_{l,u,d}$ where $\tilde{M} \equiv \frac{M}{m_3}$, m_3 being the third family mass for the corresponding flavor. The sumrule for charged lepton matrices is given by:

$$k\tilde{M}_l = r\tilde{M}_d + \tilde{M}_u \quad (14)$$

where k and r are functions of $r_{1,2}$ (given in sec. II) and fermion masses as follows:

$$k = \frac{r_2 - r_1}{4r_1r_2} \frac{m_\tau}{m_t} \quad (15)$$

$$r = -\frac{r_2 + 3r_1}{4r_1r_2} \frac{m_b}{m_t} \quad (16)$$

$$\mathcal{M}_\nu = a\left(\frac{m_b}{m_\tau}\tilde{M}_d - \tilde{M}_l\right) \quad (17)$$

These relations are valid at the B-L breaking scale v_{B-L} . The advantage of working with \tilde{M} rather than M is that the 33 elements of all $\tilde{M}_{l,u,d}$ matrices are either one or of order one; so we expect solutions for k and r also of order one. Furthermore since the formula for \mathcal{M}_ν involves only M_ℓ and M_d , $b - \tau$ unification helps to see the cancellation in the 33 element of \mathcal{M}_ν somewhat more easily. At the same time the 23 element of \mathcal{M}_ν receives only one contribution from M_ℓ since in our basis M_d is diagonal. These two results lead to atmospheric mixing angle being large [14].

To carry out the calculations, we have to solve for the two unknowns k and r using the low energy inputs from the quark and charged lepton sectors. To obtain a perturbative estimate of these parameters, we decompose $r\tilde{M}_d + \tilde{M}_u$ as:

$$\begin{pmatrix} x & 0 & 0 \\ 0 & y & \epsilon_2 \\ 0 & \epsilon_2 & z \end{pmatrix} + \begin{pmatrix} 0 & \epsilon_1 & a \\ \epsilon_1 & 0 & 0 \\ a & 0 & 0 \end{pmatrix} \equiv r \begin{pmatrix} d & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} u & \epsilon_1 & a \\ \epsilon_1 & c & \epsilon_2 \\ a & \epsilon_2 & 1 \end{pmatrix} \quad (18)$$

where $\epsilon_i, a \ll 1$ as are x and y . In this analytical approach, our procedure will be to find the eigenvalues of (18) by perturbation method and match them to the known leptonic masses at the B-L scale. The advantage of this decomposition is that it allows a nice perturbative determination of the eigenvalues analytically without having to resort to immediate numerical analysis. We will compare our results with the numerical evaluation using Mathematica.

The i^{th} eigenvalue $\lambda_i = \lambda_i^{(0)} + \lambda_i^{(2)}$ is found to be

$$\begin{aligned} \lambda_1^{(0)} &= x \\ \lambda_2^{(0)} &= \frac{y + z - \sqrt{(z - y)^2 + 4\epsilon_2^2}}{2} \\ \lambda_3^{(0)} &= \frac{y + z + \sqrt{(z - y)^2 + 4\epsilon_2^2}}{2} \sim z + \frac{\epsilon_2^2}{z} + zO(10^{-2}) \\ \lambda_2^{(2)} &\simeq \frac{(z\epsilon_1 - a\epsilon_2)^2}{\lambda_2^{(0)} z^2} \simeq O(10^{-2})\lambda_2^{(0)} \\ \lambda_3^{(2)} &\simeq \frac{a^2}{\lambda_3^{(0)}} \simeq O(10^{-2})\lambda_3^{(0)} \\ \lambda_1^{(2)} &= -(\lambda_2^{(2)} + \lambda_3^{(2)}) \end{aligned} \quad (19)$$

We consider only cases where $y \simeq 10^{-2}$ and $z > 0.1$. Within this regime, the unperturbed 2nd and 3rd lepton masses are accurate up to a few %. However, the higher order electron mass correction is big and so the perturbation formula breaks down for this case. We therefore use the perturbation technique for the second and third generation masses but use the determinant to find that for the first generation. As mentioned, we will check the validity of perturbation result using numerical methods.

Taking determinant of the above equation 18, we find that the three charged lepton masses are related as follows:

$$k^3 \tilde{m}_e \tilde{m}_\mu = xyz - x\epsilon_2^2 - ya^2 - z\epsilon_1^2 + 2a\epsilon_1\epsilon_2 \quad (20)$$

$$k\tilde{m}_\mu = \lambda_2 \simeq \lambda_2^{(0)} \quad (21)$$

$$k = \lambda_3 \simeq \lambda_3^{(0)} \simeq z + \frac{\epsilon_2^2}{z} \quad (22)$$

We now solve the above equation by substituting $x, y, z, a, \epsilon_1, \epsilon_2$ with the corresponding elements in the matrix $rM_d + M_u$. From eq.(20), and find

$$k(1 + \tilde{m}_\mu) = y + z \quad (23)$$

$$k = z + \frac{\epsilon_2^2}{z} \quad (24)$$

Since Eq.18 tells us that $z = 1 + r$ and $y = rs + c$, we can use the above two equations to determine the parameters k and r , which we can then use to find neutrino masses and mixings. We find k and r to be

$$r = \frac{(s + c - 2\tilde{m}_\mu) \pm \sqrt{(s - c)^2 - 4(\tilde{m}_\mu - s)(1 + \tilde{m}_\mu)\epsilon_2^2}}{2(\tilde{m}_\mu - s)} \quad (25)$$

$$k = \frac{(1 + s)r + 1 + c}{1 + \tilde{m}_\mu}$$

and a consistency relation for the d-quark mass

$$d = \frac{k^3 \tilde{m}_e \tilde{m}_\mu + z\epsilon_1^2 + ya^2 - 2a\epsilon_1\epsilon_2 - u(yz - \epsilon_2^2)}{r(yz - \epsilon_2^2)} \quad (26)$$

In order to get a rough feeling for the way the maximal neutrino mixings arise, let us diagonalize the charged lepton mass matrix given in Eq. 14 and write the neutrino mass matrix in this basis:

$$\mathcal{M}_\nu = a \left(\frac{m_b}{m_\tau} U_l^\dagger \tilde{M}_d U_\ell - \tilde{M}_l^D \right) \quad (27)$$

Where \tilde{M}_l^D is the diagonal charged lepton mass matrix with τ mass is 1. U_l is the rotation matrix diagonalize charged lepton mass. U_l can be written approximately as

$$U_l \simeq \begin{pmatrix} 1 & \delta_1 & \delta_2 \\ \Delta_1 & \cos \phi & \sin \phi \\ \Delta_2 & -\sin \phi & \cos \phi \end{pmatrix}, \quad (28)$$

where

$$\tan \phi = \frac{\epsilon_2}{z - \lambda_2^{(0)}}. \quad (29)$$

The parameters δ_i and Δ_i are given to lowest order in perturbation theory by

$$\begin{aligned} \delta_1 &= \frac{\epsilon_1 \cos \phi - a \sin \phi}{k\tilde{m}_\mu - x} \\ \delta_2 &= \frac{\epsilon_1 \sin \phi + a \cos \phi}{k - x} \\ \Delta_1 &= -\delta_1 \cos \phi - \delta_2 \sin \phi \\ \Delta_2 &= \delta_1 \sin \phi - \delta_2 \cos \phi \end{aligned} \quad (30)$$

Using these parameters and neglecting small terms due to δ_1 and δ_2 multiplying light quark masses, we find that

$$M_\nu \simeq \begin{pmatrix} m_d - m_e + m_s \Delta_1^2 + m_b \Delta_2^2 & m_s \Delta_1 \cos \phi - m_b \Delta_2 \sin \phi & m_s \Delta_1 \sin \phi + m_b \Delta_2 \cos \phi \\ m_s \Delta_1 \cos \phi - m_b \Delta_2 \sin \phi & m_s - m_\mu + m_b \sin^2 \phi & -m_b \sin \phi \\ m_s \Delta_1 \sin \phi + m_b \Delta_2 \cos \phi & -m_b \sin \phi & -m_b \sin^2 \phi + m_b - m_\tau \end{pmatrix} \quad (31)$$

We now find the following analytic expression for the atmospheric mixing angle from Eq. 31 to leading order ignoring small terms to be:

$$\begin{aligned} \tan \theta_A &\simeq \frac{2}{q + \sqrt{q^2 + 4}} \\ q &= \frac{2m_b \sin^2 \phi + (m_\tau - m_b) + (m_s - m_\mu)}{m_b \sin \phi} \end{aligned} \quad (32)$$

For $|q| \leq 1$, we get $\sin^2 2\theta_A \geq 0.8$. We see that $b - \tau$ unification i.e. $m_b \simeq m_\tau$ and $m_b \sin \phi \simeq (m_b - m_\tau)$ are important to get a large θ_A . Also we need to have $m_s < 0$ and $m_\mu > 0$.

V. PREDICTIONS FOR NEUTRINO MASSES AND MIXINGS

In order to obtain the predictions for neutrino masses and mixings in our model, we will need the values of quark masses and mixings at the seesaw scale. Experiments determine these input parameters near the GeV scale and they need to be extrapolated to the B-L scale which is near 10^{15} - 10^{16} GeV where our Equation (6) is valid. Taking the values for the quark masses and mixings at the B-L scale we can determine k and r approximately. We will use this determination of k and r to solve for neutrino masses and mixings using the relation in Eq.17. We will also compare our results with a direct numerical scan of the Eq. 14 i.e. not using perturbation method to obtain k and r . Results obtained by both methods are in agreement.

In our model, the theory below the B-L breaking scale is the MSSM whose effect on fermion mass extrapolation is a well studied problem [18]. We will use the two loop analysis in the paper by Das and Parida [18] in our analysis. Our strategy will be to take the values of the quark masses at the scale $v_{B-L} \simeq 10^{16}$ GeV given in [18]. In Table I, we give the input values of masses and mixings for values of the MSSM parameter $\tan\beta = 10$ and 55.

| input observable | $\tan\beta = 10$ | $\tan\beta = 55$ |
|------------------|-------------------------------|------------------------------|
| m_u (MeV) | $0.72^{+0.13}_{-0.14}$ | $0.72^{+0.12}_{-0.14}$ |
| m_c (MeV) | $210.32^{+19.00}_{-21.22}$ | $210.50^{+15.10}_{-21.15}$ |
| m_t (GeV) | $82.43^{+30.26}_{-14.76}$ | $95.14^{+69.28}_{-20.65}$ |
| m_d (MeV) | $1.50^{+0.42}_{-0.23}$ | $1.49^{+0.41}_{-0.22}$ |
| m_s (MeV) | $29.94^{+4.30}_{-4.54}$ | $29.81^{+4.17}_{-4.49}$ |
| m_b (GeV) | $1.06^{+0.14}_{-0.08}$ | $1.41^{+0.48}_{-0.19}$ |
| m_e (MeV) | 0.3585 | 0.3565 |
| m_μ (MeV) | $75.6715^{+0.0578}_{-0.0501}$ | $75.2938^{+1912}_{-0.0515}$ |
| m_τ (GeV) | $1.2922^{+0.0013}_{-0.0012}$ | $1.6292^{+0.0443}_{-0.0294}$ |

Table I: The extrapolated values of quark and lepton masses at the GUT scale from the last reference in [18]. We have kept the errors to only two significant figures in the quark masses.

For the mixing angles at GUT scale, we take:

$$V_{CKM} = \begin{pmatrix} 0.974836 & 0.222899 & -0.00319129 \\ -0.222638 & 0.974217 & 0.0365224 \\ 0.0112498 & -0.0348928 & 0.999328 \end{pmatrix} \quad (33)$$

In the first perturbative method, we use the above input values to obtain k and r using Eq. 25 and search for values around them that give a good fit to charged lepton masses and then use them in Eq.17 to derive the neutrino masses and the three mixing angles: $\sin^2 2\theta_\odot$, $\sin^2 2\theta_A$ and U_{e3} . The best fit range for k, r are $-.78 \leq r \leq -.74$ and $0.23 \leq k \leq .26$. We also do a direct numerical solution. Both the results are in agreement. (We ignore CP violation in this work.)

Note that the sign of a fermion is not physical, which leads to several choices for the sign of fermion masses that we have put into our search for solutions. The solutions we present here correspond to $m_{e,\mu,\tau,b,t} > 0$ and $m_{c,d,s} < 0$ upto an overall sign.

Our results are displayed in Fig. 1-3 for the case of the supersymmetry parameter $\tan\beta = 10$. In these figures, we have restricted ourselves to the range of quark masses for which the atmospheric mixing angle $\sin^2 2\theta_A \geq 0.8$. (For presently preferred range of values of $\sin^2 2\theta_A$ from experiments, see [19]). We then present the predictions for $\sin^2 2\theta_\odot$, Δm_\odot^2 and U_{e3} for the allowed range $\sin^2 2\theta_A$ in Fig.1, 2 and 3 respectively. The spread in the predictions come from uncertainties in the s, c and the b -quark masses. Note two important predictions: (i) $\sin^2 2\theta_\odot \geq 0.91$ and $U_{e3} \sim \pm 0.16$. The present allowed range for the solar mixing angle is $0.7 \leq \sin^2 2\theta_\odot \leq 0.99$ at 3σ level [19,20]. The solutions for the neutrino mixing angles are sensitive to the b quark mass.

It is important to note that this model predicts the U_{e3} value very close to the present experimentally allowed upper limit and can therefore be tested in the planned long base line experiments which are expected to probe U_{e3} down to the level of ~ 0.05 [21,22]. Our model would also prefer a value of the $\sin^2 2\theta_A$ below 0.9, which can also be used to test the model. For instance, the JHF-Kamioka neutrino experiment [22] is projecting a possible accuracy in the measurement of $\sin^2 2\theta_A$ down to the level of 0.01 and can provide a test of this model.

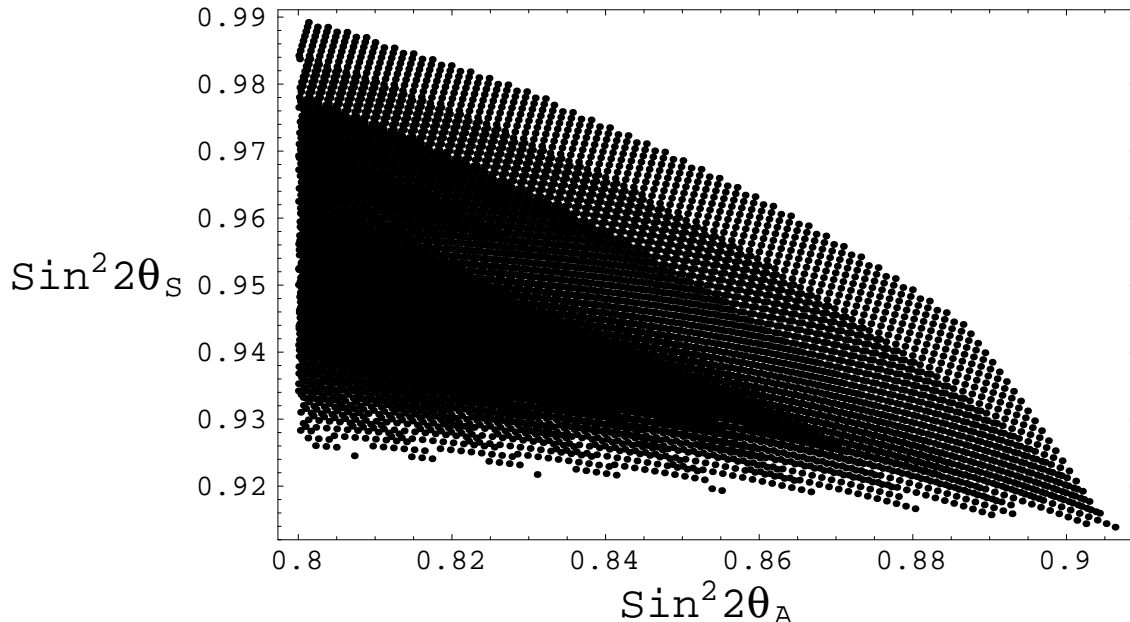


FIG. 1. The figure shows the predictions for $\sin^2 2\theta_\odot$ and $\sin^2 2\theta_A$ for the range of quark masses in table I. Note that $\sin^2 2\theta_\odot \geq 0.9$ and $\sin^2 2\theta_A \leq 0.9$

As $\tan\beta$ increases, the allowed values for the neutrino mixings and masses fall into an even narrower range.

VI. CONCLUSION

In summary, we find that a minimal SO(10) model with single **10** and **126** Higgs coupling to matter is a completely predictive model for neutrino masses and can provide an excellent description of the presently favored patterns for neutrino masses and mixings required by data. The only assumption needed is that the $SU(2)_L$ triplet vev dominates the neutrino masses. No global symmetries are invoked to generate the neutrino mass pattern, unlike most models that employ the **16** Higgs to break the B-L symmetry. The model predicts a hierarchical mass pattern for neutrinos and a value of $U_{e3} \simeq 0.16$, both of which can be tested in upcoming long baseline neutrino experiments. The atmospheric mixing angle is found to be between 0.8 and 0.9 which is also a testable prediction of the model. In our

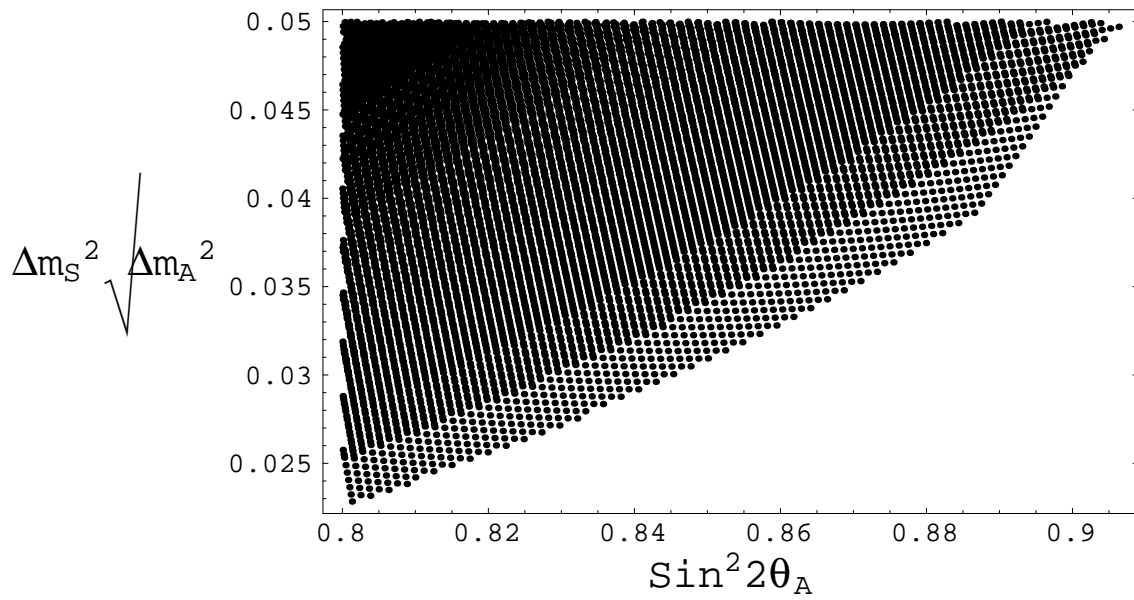


FIG. 2. The figure shows the predictions for $\sin^2 2\theta_A$ and $\Delta m_{\odot}^2 / \Delta m_A^2$ for the range of quark masses and mixings that fit charged lepton masses.

model, the Yukawa matrices have a hierarchical pattern, a rough understanding of which could come from introducing a local horizontal $U(2)_H$ symmetry under which the first two families transform as a doublet. This and other aspects of the model such as the inclusion of a CP phase are currently under investigation.

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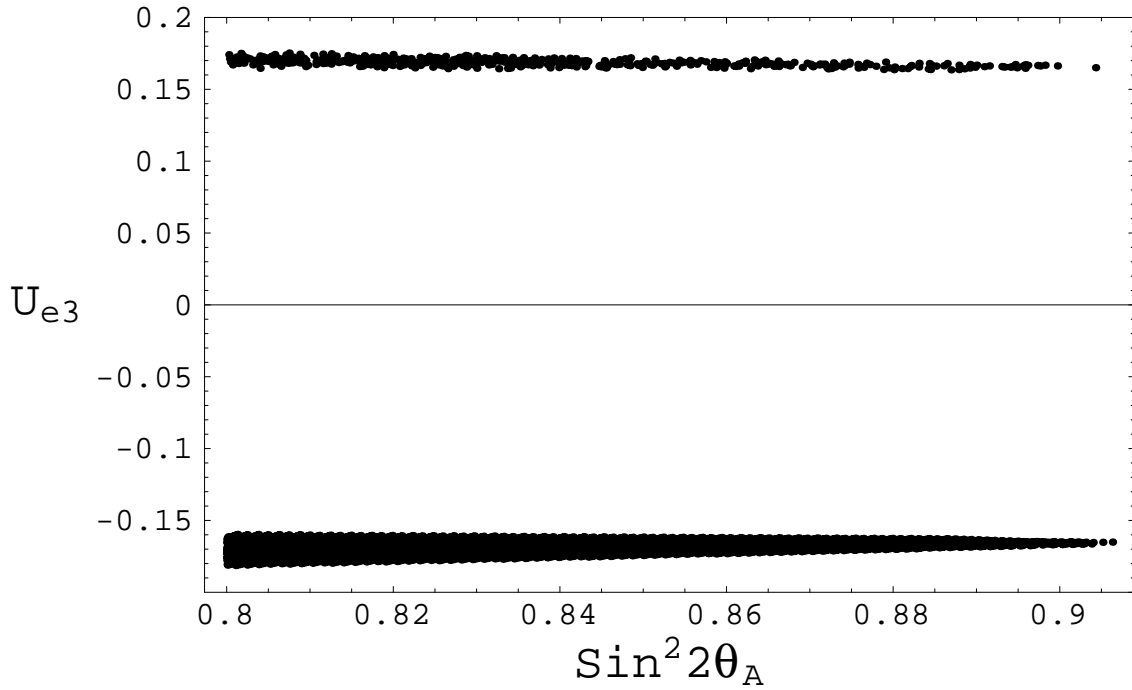


FIG. 3. The figure shows the predictions of the model for $\sin^2 2\theta_A$ and U_{e3} for the allowed range of parameters in the model. Note that U_{e3} is very close to the upper limit allowed by the existing reactor experiments.

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